DETERMINATION OF DISTRIBUTION FUNCTION USED IN MONTE CARLO SIMULATION ON SAFETY ANALYSIS OF HYDROGEN PRESSURE VESSEL

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ABSTRACT

The test data of static burst strength and load cycle strength of composite pressure vessels are often described by GAUSSian normal or WEIBULL distribution function to perform safety analyses. The goodness of assumed distribution function plays a significant role in the inferential statistics to predict the population properties by using limited test data. Often, GAUSSian and WEIBULL probability nets are empirical methods used to validate the distribution function; Anderson-Darling and Kolmogorov-Smirnov tests are the mostly favorable approaches for Goodness of Fit. However, the different approaches used to determine the parameters of distribution function lead mostly to different conclusions for safety assessments.

In this study, six different methods are investigated to show the variations on the rates for accepting the composite pressure vessels according to GTR No. 13 life test procedure. The six methods are: a) Norm-Log based method, b) Least squares regression, c) Weighted least squares regression, d) A linear approach based on good linear unbiased estimators, e) Maximum likelihood estimation and f) The method of moments estimation. In addition, various approaches of ranking function are considered. In the study, Monte Carlo simulations are conducted to generate basic populations based on the distribution functions which are determined using different methods. Then the samples are extracted randomly from a population and evaluated to obtain acceptance rate. Here, the “populations” and “samples” are corresponding to the burst strength or load cycle strength of the pressure vessels made from composite material and a plastic liner (type 4) for the storage of hydrogen. To the end, the results are discussed, and the best reliable methods are proposed.

1. INTRODUCTION

Currently, the Phase 2 of the Global Technical Regulation (GTR) on hydrogen vehicles is running. While in the 1st phase the safety value of composite pressure vessels (CPV) for hydrogen was based on a minimum burst pressure of 2.25 nominal working pressure (NWP) for carbon fibres, ECE R 134 [1], the re-discussion launched the idea of having just 2.0 NWP for carbon fibre. Since this means an effective safety factor of 1.6 in relation to the usually created maximum pressure additional measures for the assessment of the resulting safety level are needed. One of this is the assessment of the distribution functions, which describe the real statistical behaviour of strength properties.

Precisely to predict burst strength and load cycle strength of composite pressure vessels (CPV) is a key step to ensure a safe operation of hydrogen storage systems. The determination of burst pressure and fatigue life are often done by burst test and load cycle test, respectively. The results of both tests need statistical assessment for the precise description of the strength. As know from previous study, [2], the sample size and scatter distribution were identified as major influences on the uncertainty; also, distribution functions used to describe the test samples play a role for predicting a reliable survival rate for a production group, [4]. The mismatch was identified between using Normal distribution and Weibull distribution assumption from previous study, [4], for the same pressure level, it results in a SR = 99.99 % for normal distribution (ND), but only approximately survival rate, SR = 99 % for Weibull distribution (WD), see Figure 1.

Currently, safety for CPVs is regulated by applying global safety factors and testing a single CPV or very small samples of CPVs for a defined minimum strength. Two major disadvantages of this practice were elaborated. At first, statistically safe design types with low scatter of strength might be rejected based on its low strength in comparison with the general safety factors. Also, unsafe design types might get approved, just by "luckily" picking strong specimens for tests. BAM (Federal Institute of Materials Research and Testing) has proposed a probabilistic approach for gas cylinders depending on their risk potential [3] based on survival rates (SR) of at least $1 - 10^{-4}$ (9.99 %) to $1 - 10^{-8}$ (99.999999 %) at maximum service loads. This can be determined with respect to static load (time to failure/burst) or respect to load cycle to failure. However, in the very most cases the desired SRs cannot be proven with experiments due to small sample sizes. The direct experimental proof of a SR of 99 % would require more than 50 test specimens; proving SR = 99.9 % would already require more than 300 test specimens according to ranking function proposed by M. Hueck [5]. The parameters of the distribution function are estimated based on sample results; then the method of Monte-Carlo simulation is applied to generate the large numerical test data possessing the properties of burst strength or load cycle strength to extrapolate the SRs, the process of Monte-Carlo simulation (MCS) is explained in section 3.2. As an example, the acceptance rate of fatigue life cycle is analysed by operating Monte-Carlo simulations over the available range of production properties [7] to evaluate the differences of regulation and standards for composite cylinder for hydrogen, according to a deterministic requirement of GTR 13, a sample is accepted if all three of the individual test results are above the minimum required value.

In [4], G. W. Mair et al. investigated the applications of using different distributions like Log-Normal or WEIBULL to describe the load cycle strength properties of pressure vessels. It concludes that due to limited test data, the Weibull distribution as the conservative method is preferred, [6]. However, the scale factor and shape factors used in the study in [4,7] are derived from Log-Normal distribution, (this method will be explained in the next section). It is unclear among various estimation methods if any is the best method to fit the test data, and what are the impacts to the survival rate and the acceptance rate by using different methods.

It is uncomplicated to define the density function and distribution function for ND if the mean value and standard deviation of the test data are available. However, the parameters of Weibull distribution need to be determined using the estimation methods based on the test data. Nwobi and Ugomma, [8] reviewed the maximum likelihood estimation, the method of moments and the least squares method. These methods are compared, using the mean square error and the maximum likelihood criteria. The results show that the maximum likelihood estimation method significantly outperformed other methods; and the mean rank is the best method among the methods in the graphical and analytical procedures. The data used for this study is the weekly stock prices (N = 100 weeks). Similarly, Kyriaki Corinna Datsiou and Mauro Overend, [9] examined the six different methods for fitting data and estimating the Parameters of Weibull distribution based on a data of glass brittle material. They found that the weighted least squares regression is the most effective fitting method for the analysis of small samples (sample size about 10 to 20) of glass strength data. Also, in their study, the probability estimators, also called ranking functions, commonly used in Weibull distribution are evaluated. However, their evaluations are limited to the glass static strength data, the conclusion cannot be directly applied to evaluate the properties of CPVs.

Hai-Lin Lu1, Chong-Hong Chen and Jong-Wuu Wu, [10] performed a study on Weighted Least-squares Estimation of the shape parameter of the Weibull Distribution. The weight functions from Bergman, [11], and Faucher and Tyson (F&T), [12] are used in the study, their results show that the weighting method of Bergman is not significantly different from that of F&T.

Even some previous studies have been conducted with focus on the methods to estimate parameters of Weibull distribution; however, there is no clear preferred methods can be identified. Also, it is unknown how the survival rate and acceptance rate on burst strength or life cycle will be affected. Finally, this all lead to the question, what are the best methods for the assessment of data with respect to the evaluation of the best fit function.

In this study, section 2 introduces the different methods used to determine the parameters of Weibull distribution; section 3 and section 4 show the validations of different method with MCS and real test data, respectively, section 5 compares the results from test and MCS, section 6 evaluated the impact to the survival rate and acceptance rate, and section 7 is the summary and conclusions. To the best of authors' knowledge, the study conducted here is the first to use real strength data of CPVs and combined with Monte-Carlo simulation to evaluate many different methods to determine the Weibull distribution parameters in conjunction with ranking functions. The conclusions are valuable for the researchers and engineers who use statistical methods in hydrogen industry based on small test samples.

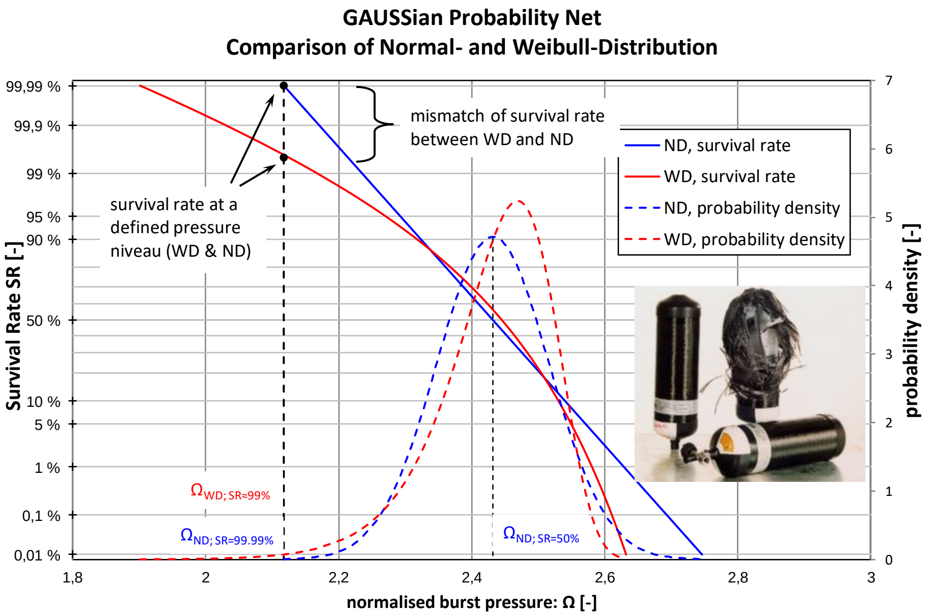

Figure 1: Differences between extrapolated survival rates for normal and Weibull distribution within the Gaussian probability net, [4].

2. METHODS TO ESTIMATE THE PARAMETERS OF WEIBULL DISTRIBUTION

2.1 Weibull distribution and ranking function

The Weibull distribution is one of the most widely used lifetime distributions in reliability engineering. It is often used to describe the fatigue life strength of material or structures. Two parameters Weibull distribution function is concerned in this study for the simplicity. It is assumed that the pressure vessels are failure free in the initial stage of the testing ($T_0 = 0$).

Density function:

$$f(x) = \frac{b}{T}\left(\frac{x}{T}\right)^{b-1} e^{-\left(\frac{x}{T}\right)^b};\ x, T, b \geq 0 \tag{1}$$

Distribution function:

$$P(x) = 1 - e^{-\left(\frac{x}{T}\right)^b};\ x, T, b \geq 0 \tag{2}$$

where T – scale factor, b – shape factor.

The distribution function can be expressed as a linearized form, see Eq. (3). Based on the linearized expression, the assumption of Weibull distribution can be checked using Weibull distribution net. Also, the scale parameter and shape factor can be read from the graphic expression.

$$\ln\left(\ln\left(\frac{1}{1-P(x)}\right)\right) = b * \ln x - b * \ln T \tag{3}$$

$P_{(x)}$ is the probability which can be determined using ranking function. The general form of ranking function:

$$R_j = \frac{i-C_j}{n+1-2C_j} \tag{4}$$

Where C_j is a constant $0 \leq C_j \leq 1$, i is the index in ascending order and n is the sample size. According to [9], followings are the common ranking functions used in Weibull distribution.

Mean rank:

$$C_1 = 0, then\ R_1 = \frac{i}{n+1} \tag{5a}$$

Hazen's:

$$C_2 = 0.5, then\ R_2 = \frac{i-0.5}{n} \tag{5b}$$

Median rank:

$$C_3 = 0.3, then\ R_3 = \frac{i-0.3}{n+0.4} \tag{5c}$$

Small sample:

$$C_4 = 0.375, then\ R_4 = \frac{i-0.375}{n+0.25} \tag{5d}$$

2.2 Methods for estimation of Weibull distribution parameters

In this study, the performance of following methods for estimating the scale factor and shape factor of Weibull distribution is investigated.

a) Log-Normal based method (L-N)

The test data are evaluated using log-normal distribution, then mean value and standard deviation of normal distribution is transferred into parameters of Weibull distribution parameters T and b.

The determination of the distribution function is based on the conversion of the parameters of a log-normal distribution into a Weibull distribution according to [13].

Calculation of mean m_{logX} and standard deviation s_{logX} of a sample based on the log normal distribution:

$$m_{log\,X} = \frac{1}{n} \sum_{i=1}^{n} log(X_i) \tag{6}$$

$$s_{log\,X} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(\log(X_i) - m_{log\,X}\right)^2} \tag{7}$$

The conversion of the parameters of a log normal distribution (Equation 6 and 7) into the Weibull parameters T and b can be carried out according to [13] as follows:

$$b = 1.02743/\left(2 \cdot s_{logX}\right) \tag{8}$$

$$T = 10^{m_{logX}} \cdot 10^{0,48416 \cdot s_{logX}} \tag{9}$$

This is realized by a transformation of the parameters from a Log-Normal distribution into a Weibull distribution. This transformation is based on two reference points (SR=16%; SR=84%) as shown in Figure 7 in [6].

b) Least squares regression (LR)

The Weibull parameters are determined in the Least Squares Regression method by minimizing the sum of squared residuals of the x values about Eq. (3).

$$b = \frac{n\sum_{i=1}^{n}[ln(x_i)*y_i] - \sum_{i=1}^{n}(lnx_i)\sum_{i=1}^{n}(y_i)}{n\sum_{i=1}^{n}[(lnx_i)^2] - \left[\sum_{i=1}^{n}(lnx_i)\right]^2} \tag{10}$$

$$-b * \ln(T) = \frac{\sum_{i=1}^{n}(y_i) - b*\sum_{i=1}^{n}(lnx_i)}{n} \tag{11}$$

$$y = \ln\left(\ln\left(\frac{1}{1-P(x)}\right)\right) = b * \ln x - b * \ln T \tag{12}$$

LR provides a biased estimation since the same weight to each data point is applied.

c) Weighted least squares regression (WLR)

WLR introduces the weight factors into LR based on the uncertainty of y and Pi.

$$b = \frac{\sum_{i=1}^{n} W_i * \sum_{i=1}^{n}[ln(x_i)*y_i*W_i] - \sum_{i=1}^{n}[(lnx_i)*W_i]*\sum_{i=1}^{n}(y_i*W_i)}{\sum_{i=1}^{n} W_i * \sum_{i=1}^{n}[(lnx_i)^2*W_i] - \left[\sum_{i=1}^{n}(lnx_i)*W_i\right]^2} \tag{13}$$

$$-b * \ln(T) = \frac{\sum_{i=1}^{n}(y_i*W_i) - b*\sum_{i=1}^{n}[(lnx_i)*W_i]}{\sum_{i=1}^{n} W_i} \tag{14}$$

The weight factor W_i is calculated based on the method proposed by Bergman, [14], shown in Eq. (15).

$$W_i = [(1 - P_i) * \ln(1 - P_i)]^2 \qquad (15)$$

According to [10], no significant difference is identified by using weight factor from Bergman [14], and F&T, only Eq. (15) as weight function is employed in the study.

d) A linear approach based on good linear unbiased estimators (GLUE)

GLUE is a simple unbiased method to determine the shape and scale factor, this method is proposed by Bain, [16,17] and is prescribed in EN 12603:2003, [18]. An unbiasing constant, kn, and an integer number, s, are used in the calculation. Where kn is given in tables in EN12603:2003 for n=2-60 [18], and s is the integer number for the product of 0,84*n.

$$b = \frac{n * k_n}{\frac{s}{n-s}\sum_{i=s+1}^{n}(lnx_i) - \sum_{i=1}^{s}(lnx_i)} \qquad (16)$$

$$T = \exp(\frac{1}{n}\sum_{i=1}^{n}(lnx_i) + 0.5772\frac{1}{b}) \qquad (17)$$

e) Maximum likelihood estimation (MLE)

This method is prescribed in ASTM C1239-13 [19] and DIN EN 843-5 [20]. The likelihood function L is as Eq. (18).

$$L = \prod_{i=1}^{n} f(x_i; b, T) \qquad (18)$$

The logarithm of the likelihood function is maximised by differentiating ln(L) over each of the unknown parameters b and T), and subsequently setting each of the partial derivatives to 0. Iterative numerical methods are used to obtain estimates for b and T.

$$\frac{\partial \sum_{i=1}^{n} lnL}{\partial b}=0 \text{ and } \frac{\partial \sum_{i=1}^{n} lnL}{\partial T}=0 \qquad (19)$$

$$\frac{n}{b} - n\frac{\sum_{i=1}^{n}[x_i^b * lnx_i]}{\sum_{i=1}^{n} x_i^b} + \sum_{i=1}^{n} lnx_i = 0 \qquad (20)$$

$$T = \left(\frac{\sum_{i=1}^{n} x_i^b}{n}\right)^{1/b} \qquad (21)$$

f) The method of moments estimation (MME)

The method of moments was introduced by Pafnuty Chebyshev in 1887. The method of moments starts by expressing the population moments (i.e., the expected values of powers of the random variable under consideration) as functions of the parameters of interest. Those expressions are then set equal to the sample moments. The number of such equations is the same as the number of parameters to be estimated. Those equations are then solved for the parameters of interest. The solutions are estimates of those parameters. The parameters of the Weibull distribution can be determined from the variance and the expected value of the Weibull distribution. The expected value and the variance are expressed in Eq. (22) and (23) respectively.

$$E(X) = \frac{1}{\lambda}\Gamma(1 + \frac{1}{b})) \qquad (22)$$

$$Var(X) = \frac{1}{\lambda^2}[\Gamma(1 + \frac{2}{b})) - \Gamma^2(1 + \frac{1}{b}))] \qquad (23)$$

From Eq. (22) and (23) the Eq. (24) and (25) can be obtained.

$$u = \frac{1}{\lambda}\Gamma(1 + \frac{1}{b}) \qquad (24)$$

$$\sigma^2 = \frac{1}{\lambda^2}[\Gamma(1+\frac{2}{b})) - \Gamma^2(1+\frac{1}{b}))] \quad (25)$$

Replacing λ in Eq. (23) by using Eq. (24), the Eq. (26) is obtained.

$$\sigma^2 \quad = \frac{\mu^2}{\Gamma^2(1+\frac{1}{b})}[\Gamma(1+\frac{2}{b}) - \Gamma^2(1+\frac{1}{b})] \quad (26)$$

The equation (26) contains the gamma function, it can only be solved with iterative numerical method, e.g. Newton method to receive the shape factor b. The problem can be solved by finding the root of h(k) in Eq. (27) which is generated by subtracting of σ^2 from Eq. (26).

$$h(k) \quad = \mu^2[\frac{\Gamma(1+\frac{2}{b})}{\Gamma^2(1+\frac{1}{b})} - 1] - \sigma^2 \quad (27)$$

3. EVALUATION OF METHODS USING MONTO-CARLO SIMULATION

3.1 Goodness of Fit test

The assumption of distribution function based on a set of test data has a serious risk if it is not validated. Two approaches are often employed to perform the check of assumed distribution functions. One is the empirical procedure and the other one is a statistical method for assessing the distribution of a data set, Goodness of Fit. For the empirical method, the Gaussian probability net, Weibull distribution net are preferred. There are various methods of Goodness-of-Fit, namely Anderson-Darling (AD), Kolmogorov-Smirnov (KS) tests, Kuiper's tests and Shapiro–Wilk test. Both the AD and KS GoF tests use the cumulative distribution function approach and therefore belong to the class of "distance tests". For Goodness of Fit, the Anderson-Darling (AD) test is the mostly preferred approach.

The shape factor and character factor can be determined using the methods introduced in section 2. In this study, Anderson darling test is performed to check the Goodness-Of-Fit of data from real test or Monte-Carlo simulation.

The AD test for Weibull distribution is explained in detail in [21,22] and is defined by:

$$AD = \sum_{i=1}^{n} \frac{1-2i}{n}\{\ln(1 - exp[-Z_{(i)}]) + Z_{(n+1-i)}\} - n \quad (28)$$

$$AD^* = \left(1 + \frac{0.2}{\sqrt{n}}\right) * AD \quad (29)$$

Where $Z_{(i)} = [x_{(i)}/T]^b$, T and b are the scale factor and shape factor, respectively. The OSL (observed significance level) probability is used for quantifying the parameters of Weibull distribution. If OSL < 0.05 then the Weibull assumption is rejected, and the error committed is less than 5%. The OSL formula is given by:

$$OSL = 1/\{1 + \exp[-0.1 + 1.24 * \ln(AD^*) + 4.48(AD^*)]\} \quad (30)$$

3.2 Monte-Carlo simulation

In Monte-Carlo simulation (MCS), the random numbers are generated, the random numbers are corresponding to the probability, P_i of survives rate expressed in the Eq. (31).

$$1-P_i=1-e^{-\left(\frac{N}{T}\right)^b} \quad (31)$$

Equation (31) can be transformed into N (fatigue life cycle as random variable) in the following steps in a more suitable form (equation 31a):

$\ln(P_i) = -1 \cdot \left(\frac{N}{T}\right)^b$ (31a)

$N = \left(-\ln(P_i)\right)^{\frac{1}{b}} * T$ (31b)

In Monte-Carlo simulation, the random numbers Pi (probability) are generated using Excel Tool in this study, then the data of mechanical properties such as fatigue life strength or burst strength can be generated from equation (31b). The scale parameter T and shape factor b are calculated using estimation methods introduced in section 2 based on test data.

3.3 Results evaluation

Numerical test data corresponding to the fatigue life strength of pressure vessels are generated using Monte-Carlo Simulation. The scale factor, T = 48186, and shape factor b = 4.281, are applied to equation (31f) to produce fatigue life cycle data. The values of scale factor and shape factor are estimated from a real fatigue life test of composite pressure vessels. (In fact, T and b can be arbitrary). At first, 100,000 fatigue life data are generated as a population of the pressure vessels by using Monte-Carlo simulation; subsequently, the numbers of samples in size of 10, 20, 30 40, 50 and 60, respectively, are drawn randomly from population to construct 6 sample groups. This simulates the real situation that the certain numbers of pressure vessels are taken from the production randomly for fatigue or burst test.

To study the sample size effect, six samples are constructed according to the sample size of 10, 20, 30 40, 50 and 60. Four ranking functions shown in Eq. (5a) to (5d) are considered in conjunction with the methods of least square regression (LR) and weighted least square regression (WLR). In addition, the method of maximum likelihood estimation (MLE), the method of moments estimation (MME), linear approach based on good linear unbiased estimators (GLUE) and Log-Normal based method (L-N) are applied individually to calculate the scale parameter and shape parameter of Weibull distribution. Based on each set of data generated from MCS, the parameters T and b are calculated using different methods, respectively.

Anderson Darling test as explained in section 3.1 is applied to calculate the significance level of generated data comparing with the theoretical Weibull distribution functions. Theoretical Weibull distribution functions are determined based on the scale factors and shape factors determined using different estimation methods. The higher value of observed significance level (OSL) indicates a better fit of the assumed distribution function to the test data. The performance of estimation methods studied in this paper is quantified with the index number from 1 to 12 according to OSL. The number 1 means the best performance of method, the number 12 indicates the worst fit.

The performance indexes of different estimation methods for sample size 10, 20, 30, 40, 50 and 60 are shown in Table 1. The corresponding significance levels are illustrated in Figure 2 and Figure 3 with bar diagram combined with performance index. Method of linear regression using ranking function shown in Eq. (5a), (5b), (5c) and (5d) are abbreviated as LR_R_1, LR_R_2, LR_R_3 and LR_R_4, respectively; likewise, WLR_R_1, WLR_R_2, WLR_R_3 and WLR_R_4, mean the method of weighted linear regression using ranking function shown in Eq. (5a), (5b), (5c) and (5d). The results from estimation methods containing ranking function are shown in the Figure 2 and Figure 3 with square frame.

Table 1: Performance index of varied methods to determine the parameters of WD based on MCS

	LR_ R_1	LR_ R_2	LR_ R_3	LR_ R_4	WLR _ R_1	WLR _ R_2	WLR _ R_3	WLR _ R_4	MLE	MM E	GLU E	N-L
n10	12	3	8	6	11	7	10	9	2	1	4	5
n20	11	12	4	5	7	1	3	2	6	8	9	10
n30	3	12	7	10	11	1	4	2	6	8	5	9
n40	9	8	5	6	2	4	1	3	10	7	12	11
n50	4	9	6	7	5	1	3	2	11	10	8	12
n60	12	5	10	8	6	1	3	2	7	4	9	11

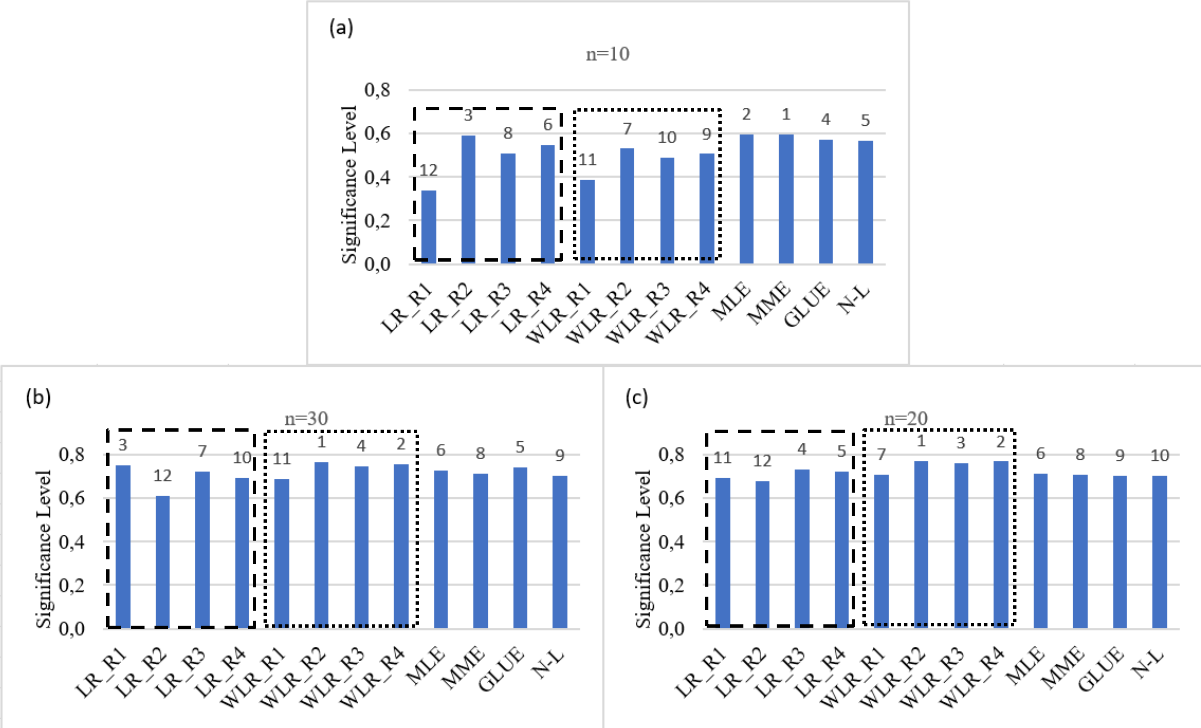

Figure 2: Significance level and performance index of estimation methods for sample size 10, 20 and 30 based on MCS

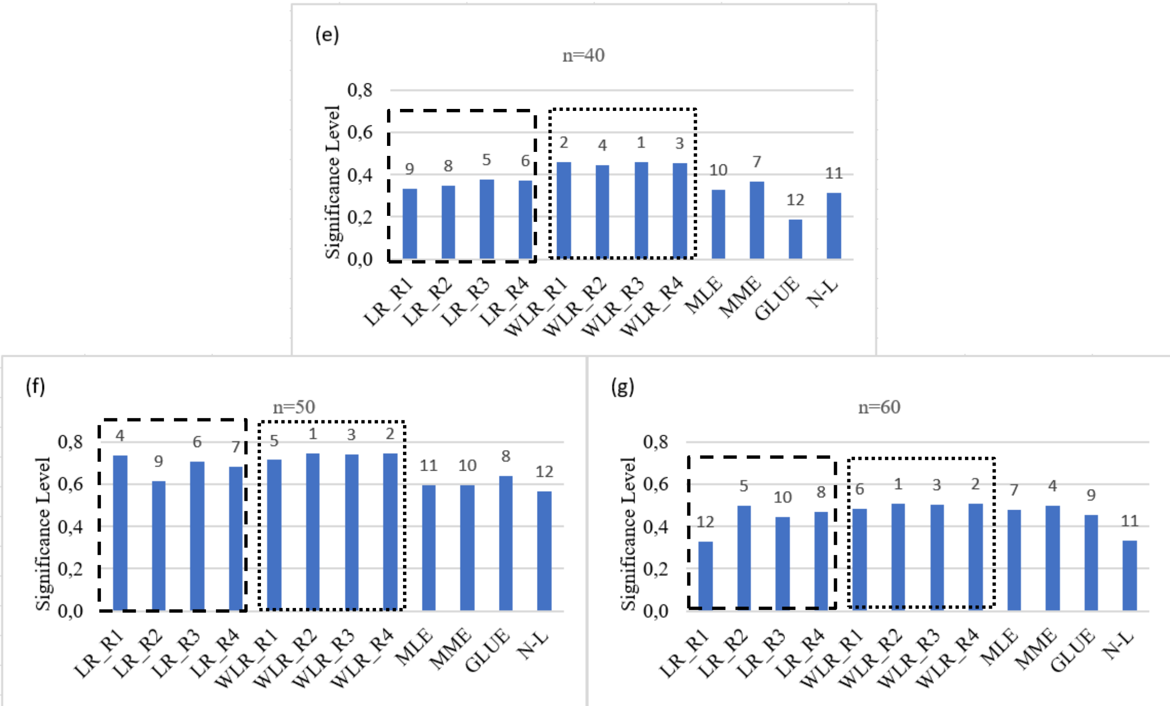

Figure 3: Significance level and performance index of estimation methods for sample size 40, 50 and 60 based on MCS

From Figure 2 and Figure 3 it can be observed that the method of weighted square regression provides the best results in general, (dotted square frame), exempt for sample size 10 (Figure 2-a). Within dotted square frame, the performance indexes for all the sample size 10, 20, 30, 50 and 60 are ranked in the order from best to worst as WLR_R_2, WLR_R_4, WLR_R_3 and WLR_R_1; similar significance levels of WLR method are shown for sample size 40 (Figure 3-e). This observation is similar to the conclusion from [9] that the weighted least square regression in conjunction with Harz's ranking function performances better but contradicts to the results from [8].

For the test data of sample size n = 10 (Figure 2-a), the MLE and MME show the better results than others; however, for the rest of sample sizes, there is no obvious advantage shown by using these two methods. The results of using different ranking functions in LR method are shown in bar diagram in "dash square frame" in Figure 2 and Figure 3, no clear trend can be identified for which ranking method in conjunction with least square regression is better.

It attempted to compare the significance levels of estimation methods between the data with different sample size; however, the data generated from MCS are randomly, and the samples are built randomly. Table 2 shows the scale factor and shape factor obtained from MCS generated data from different sample size with WLR combined with Harz's ranking. Different Weibull distribution parameters are obtained from varied sample size, even the identical scale factor and shape factor are applied to Monte-Carlo simulations. Either large numbers of sample size or high numbers of repeated small sample size are needed to obtain similar distribution function of the data. This indicates a risk that to draw a conclusion based on small sample size but apply it as general rule. In the next section, the repeated Monte-Carlo simulations are conducted to show the differences on the results which are obtained from single simulation and the repeated simulations.

Table 2: Weibull parameters determined from WLR (Harz's ranking) for different sample size

	Weighted Least Square (Harz's ranking)	
	Scale factor T	Shape factor b
n10	50095	4,029
n20	46967	4,682
n30	49170	4,501
n40	46753	4,317
n50	48821	3,592
n60	50474	4.618

3.4 Reproductivity of Monte-Carlo simulations

It has been realized that to repeat a Monte-Carlo simulation with small sample size wouldn't produce the same or similar data as the previous one. For the small sample size, the random effect dominates the significance levels for the different sample size if the sample size is not large enough or the small samples are not repeated sufficiently. The Monte-Carlo simulations are repeated for one million times based on the parameters determined from test data with sample size 37, the mean significance levels (OSLs) of using different estimation methods to fit the test data are summarized in Figure *4*; the histograms of OSL for one million repeated MCSs by using varied estimation methods and ranking functions are shown in Figure *5* as an example.

In Figure 4, the solid bars are the values of mean OSL, the right bars with dots inside show accumulative probability of occurrence with OSL above mean OSL. The order of averaged goodness of fit shown on Figure 4 is different as that from Figure 6-d with sample size 37 from a real test data. Figure 6-d shows that the best method is GLUE and the worst is the WLR_R_1; however, Figure 4 shows the best mean OSL is from MME and the worst is from LR_R_1.

The order of goodness of fit based on mean OSL of different methods is shown in Figure *4* with corresponding probability, (probability for OSL of simulation above mean OSL). It indicates that the goodness of fit regarding different estimation methods is a statistical matter. Figure *5* shows the OSLs and their probabilities for one million repeated Monte-Carlo simulations for WLR as an example. The horizontal axis in the diagrams presents OSL over one million simulations in ascending order, the vertical axis shows the corresponding probability. The histogram in Figure *5* shows the OSLs varying from extreme small values to extreme high values with decreased probability. The mean OSL is illustrated in the figure with vertical red line.

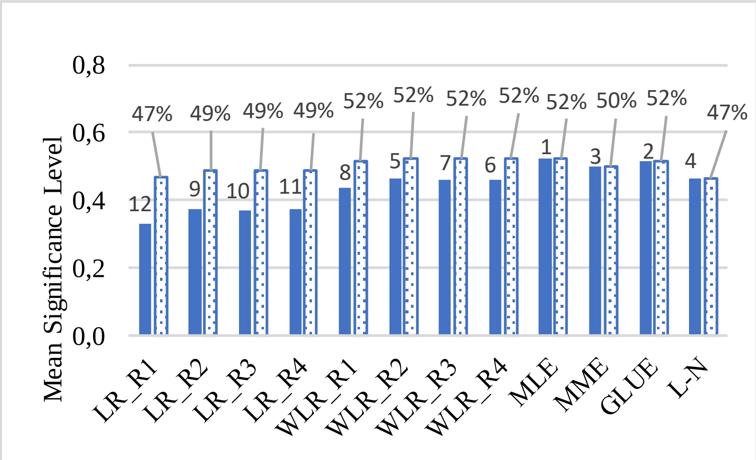

Figure 4: Mean OSL and probability of occurrence above OSL for sample size 37

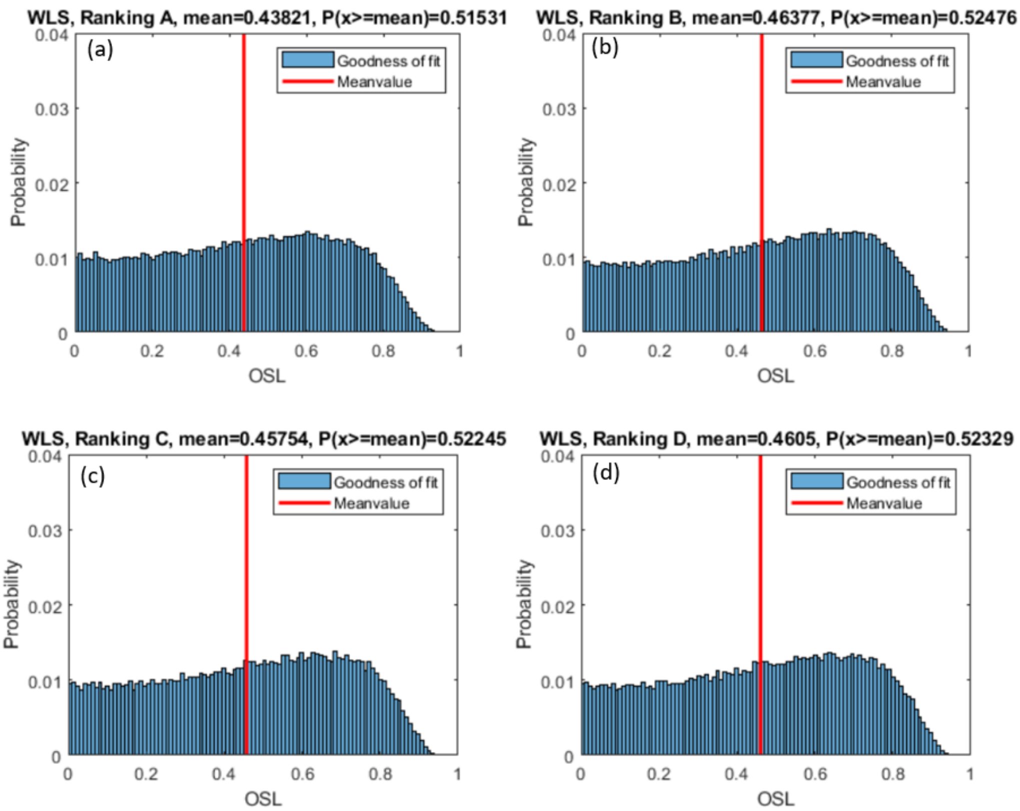

Figure 5: Mean significance level of repeated MCS based on WLR method

In reality it wouldn't be feasible to repeat the test data with high frequencies; however, the study of repeated simulations prevents the conclusion drawn wrongly from single event based on small sample size, the order of goodness of fit regarding different estimation methods is a statistical matter.

4. EVALUATION OF METHODS BASED ON TEST DATA

The real test data of composite pressure vessels are used to investigate the performance of different estimation methods. The following test data are used:

a) n = 10; type 4, GFRP cylinder, life cycle test.

b) n = 12; GFRP-type II cylinder for medical O2, [23].

c) n = 24; test results of burst pressure of type 4 CFRP cylinder, composite cylinder design types, named design D (type IV, CFRP, for breathing air, PH =45MPa), [4].

d) n = 37; breathing air cylinders of type III made from CFRP with aluminum liner for 300 bar nominal working pressure after 15 years of service at the Berlin Fire Department, [24].

The performance indexes of concerned estimation methods based on four sets of test data as shown above are recorded in Table 3; the significance levels with orders of performance indexes are shown in Figure 6. The performance indexes are calculated in the identical way as those in section 3.3. The methods of WLR_R_2, WLR_R_3 and WLR_R_4 are identified as better; even some of other methods show similar level of significance. Except for sample size 10 (Figure 6-a), the order of performance index of WLR with ranking function are consistent and identical as those obtained from Monte-Carlo simulations in section 3. No pattern can be observed from the results of LR using different ranking functions.

Table 3: Performance index of varied methods to determine the parameters of WD based on test data

	LR_ R_1	LR_ R_2	LR_ R_3	LR_ R_4	WLR _ R_1	WLR _ R_2	WLR _ R_3	WLR _ R_4	MLE	MM E	GLU E	N-L
n10	12	5	11	9	10	7	1	2	4	3	8	6
n12	11	7	3	2	12	4	9	8	10	6	1	5
n24	9	10	4	6	7	1	3	2	5	11	12	8
n37	9	10	3	5	12	2	6	4	11	8	1	7

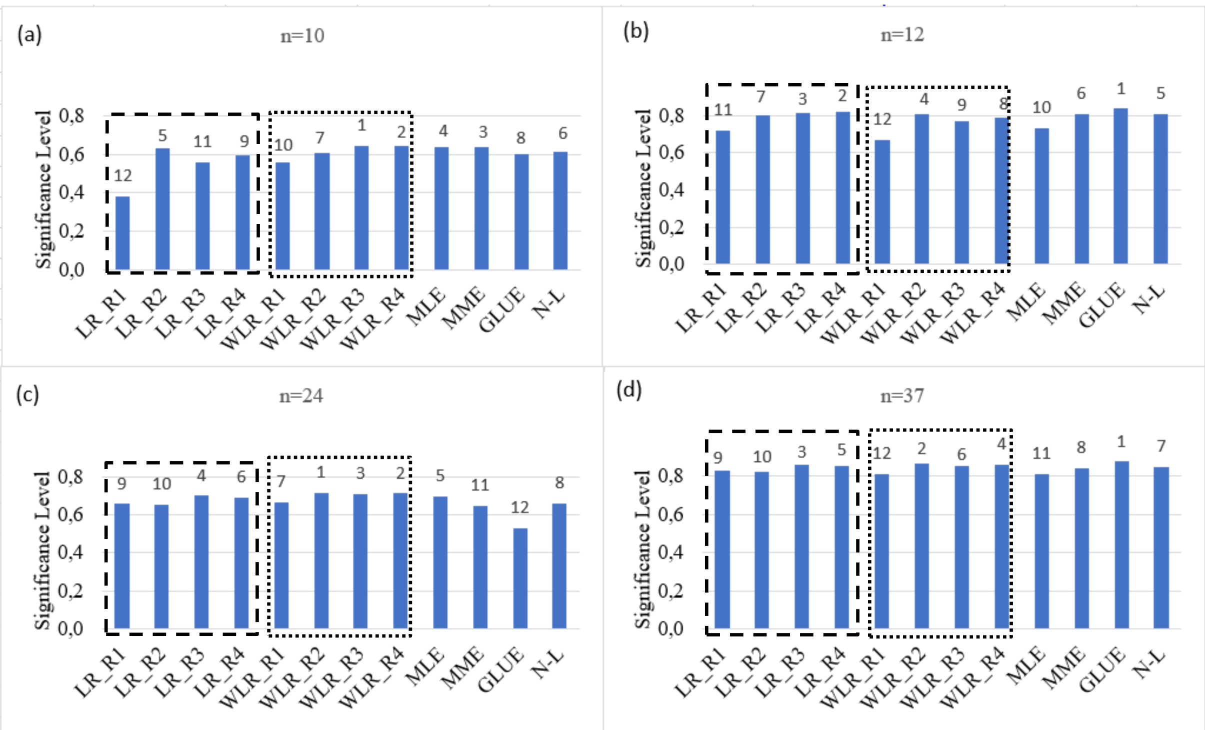

Figure 6: Significance level of varied estimation methods for sample size 10, 12, 24 and 37 based on real test data

5. COMPARISOM OF RESULTS FROM MCS AND TEST DATA

The significance levels based on sample size 10 from real test data are compared with those from MCS data with same sample size 10, see Figure 2-a and Figure 6-a. The scale factor T=48186 and shape factor b=4,281 are determined from test data with sample size 10 using Log-Normal based method, and then applied to the Monto-Carlo simulations. From the significance level and the performance order shown in Figure 2-a and Figure 6-a, no good correlation can be found between the test data and the MCS produced data, even the same Weibull parameters are used in MCS data. This confirms that the performances of different estimation methods cannot be evaluated only on the single event of test data or simulation with small sample size.

Test data with sample size n=37, Figure 6-d, is compared with Monte-Carlo simulation with sample size 40, Figure *3*-e. The significance levels are compared. The results show a good correlation except for using method of GLUE. The performance of GLUE in test data shows the best fit; however, it results a worst performance in MCS. The reason is that the method of GLUE is sensitive to the number of samples. The difference between sample size 37 and 40 results a different "s" in Eq (16), the results are sensitive to "s".

6. PROPOSED METHODS USED FOR SAFETY ASSESSMENT

The objective of this study is to find out the best method to describe the test data using distribution function. Two functions of Weibull distribution with the best and the worst extreme cases are plotted in the GAUSSian probability net to show the variations on the survival rate. Figure 7 presents the probability net for burst test with sample size 24. As known from previous study, [23], the data of burst test often normal distributed, however, the distribution of test data presented in Figure 7 can be precisely described by Weibull distribution. It results about 4% difference in terms of survival rate between using Weibull and normal distribution. To use weighted least square regression method, (the best here) in conjunction with ranking function of Harz shows difference in overall comparing to that using GLUE, (the worst); however, the variations on the prediction of maximum survival rate are small. They are all close to 95% in Figure 7.

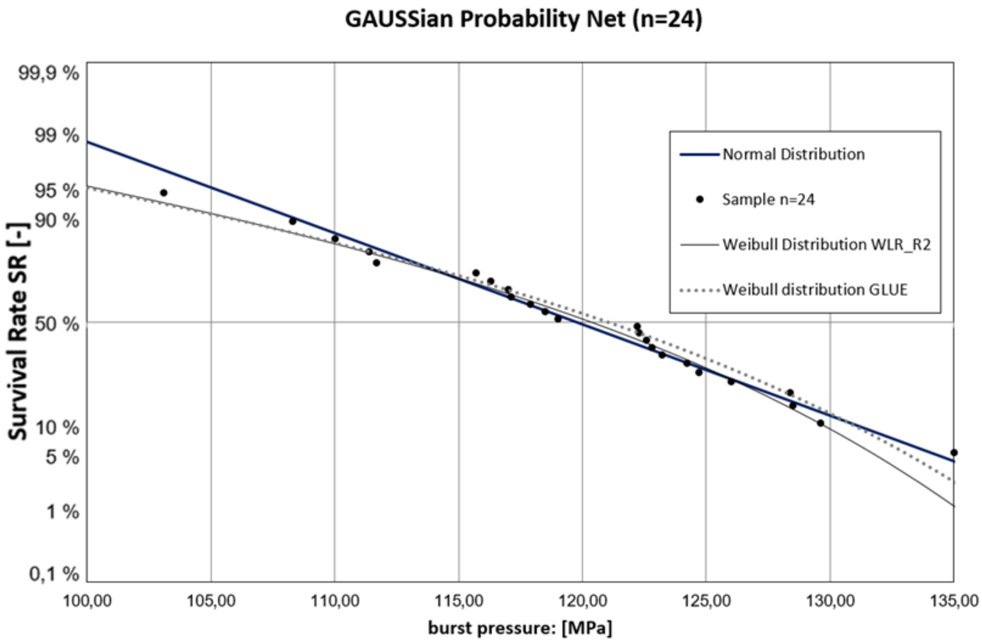

Figure 7: Comparison of survival rate for burst test data (n=24)

Further study is conducted to validate the acceptance rate by using different methods based on GTR No.

13 defined life cycle test procedure. Ten thousand samples are generated with sample size 3, the acceptance rates are calculated based on 11000 life cycles requirement according to GTR No. 13. The acceptance rates vary from 5,6% to 6,2% for breathing air cylinders after 15 years in service for using different methods, see Figure 8.

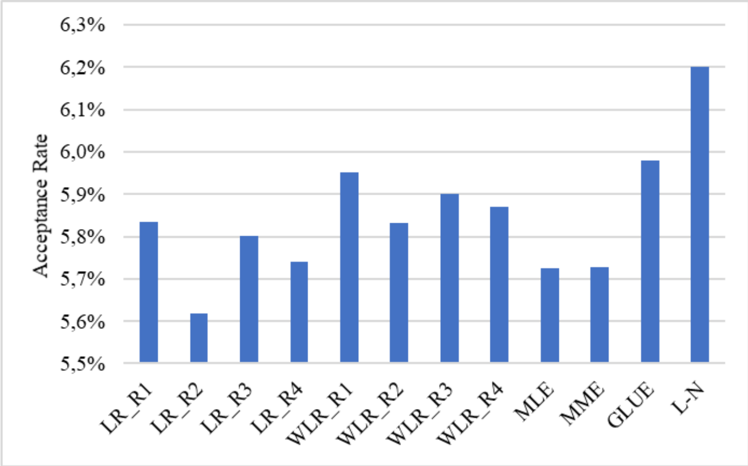

Figure 8: Different acceptance rates of life cycle according to GTR No. 13 test procedure

An example of sample with AR 6% based on estimation method GLUE is presented in the sample performance chart in Figure 9. Each dot is plotted in the diagram based on median value (vertical axis) and the scatter (horizontal axis) calculated from 3 cylinders as one sample. Black and red dots shown in Figure 9 present the samples which pass or fail the requirement respectively. Median value and scatter of the samples play a crucial role in the probabilistic approach by BAM.

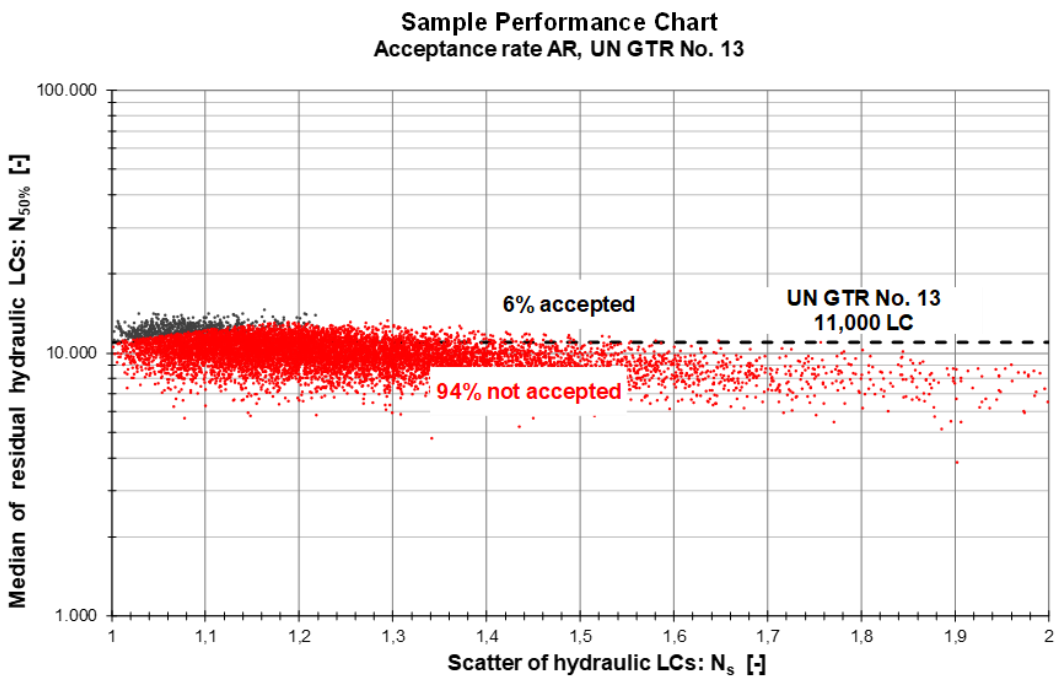

Figure 9: Sample performance chart for the samples with acceptance rate 6% of breathing air cylinders after 15 years in service

7. SUMMARY AND CONCLUSIONS

In this study, six methods used for estimating 2-parametric Weibull distribution parameters are performed. Four ranking functions are included in least square regression and weighted least square regression. The Anderson Darling goodness of fit is applied to evaluate the performance of different methods based on observed significance level (OSL). The data used for study are from real strength test and the data generated using Monte-Carlo simulation. The sample sizes are varied from 10, 12, 24 and 37 for test data and 10, 20, 30, 40, 50 and 60 for Monte-Carlo simulations data. A general trend can be observed that the weighted least square method shows the better goodness of fit except for small sample size 10. The ranking function "Harz" shows the best performance combined with WLR method. No other clear conclusion can be reached to qualify the estimation methods and the ranking functions.

The conclusions drawn from the study are based on the data produced as a single event. This matches the real situation; however, the order of goodness of fit for estimation methods is a statistical matter as demonstrated in this study. The limitation may exist to use the results from single event as a general conclusion. The best estimation method and ranking function for different test data can be identified individually using the process shown in this study.

The impacts of using different estimation methods to the survival rate and the acceptance rate of composite pressure vessels according to GTR 13 life cycle requirement are insignificant for the data concerned here; even differences of performance of estimation methods and ranking function clearly exist. However, it is still to recommend using the best estimation method in conjunction with best ranking function to minimize the uncertainty in the statistical safety assessment for composite pressure vessels.

In the reality, the test data, especially the results of life cycle test may be better described by three parameter Weibull distribution, and even combined Weibull distribution. The study carried out in this paper is based on two parameters Weibull distribution, it can be extended in the further.

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